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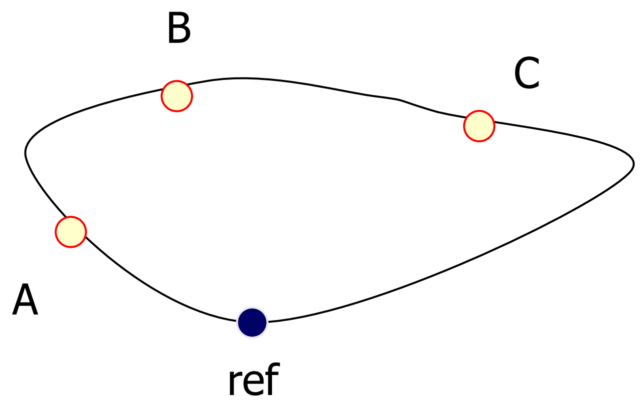
Bioelectromagnetism
Exercise #3 – Answers

Q1: Measurement Lead (Lead) Vector

- **Three electrodes (a, b and c) are on the surface of a volume conductor. Inside the conductor is a dipole source. The lead vectors of this dipole defined at the three locations (a, b and c) are:**

$$\bar{c}_a = \bar{i} + 2\bar{j} + \bar{k}$$
$$\bar{c}_b = 3\bar{i} + 7\bar{j} + 2\bar{k}$$
$$\bar{c}_c = 7\bar{i} + 5\bar{j} + 4\bar{k}$$

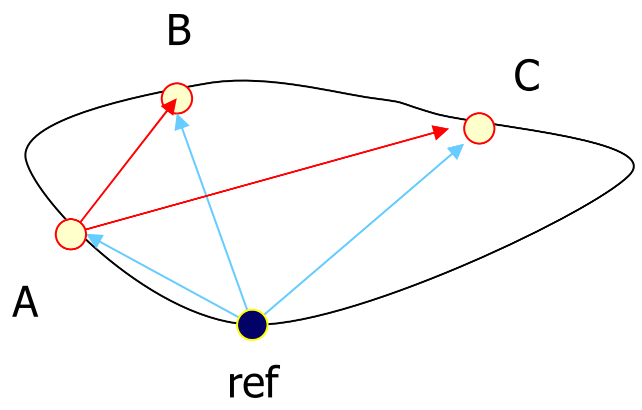
The dipole is parallel to the unit vector i. What is the ratio of the voltages measured between electrodes a and b to a and c?



Q1: Measurement Lead (Lead) Vector

- $\mathbf{c}_{ab}$ & $\mathbf{c}_{ac}$

$$\bar{c}_a = \bar{i} + 2\bar{j} + \bar{k}$$
$$\bar{c}_b = 3\bar{i} + 7\bar{j} + 2\bar{k}$$
$$\bar{c}_c = 7\bar{i} + 5\bar{j} + 4\bar{k}$$



$\mathbf{c}_{ab} = \mathbf{c}_b - \mathbf{c}_a = 2i + 5j + k$

$\mathbf{c}_{ac} = 6i + 3j + 3k$

- V_{ab} & V_{ac}

  $\mathbf{p} = xi$ (parallel to the x-axis =i)

  $V_{ab} = 2*x$ [V]

  $V_{ac} = 6*x$ [V]

- $V_{ab}/ V_{ac} = 1/3$

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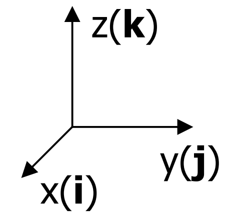
Q2: 12-lead ECG Lead Vectors

- Derive the lead vectors of the limb leads I, II, and III, leads VR, VF, VL, and the Goldberger leads aVR, aVL ja aVF in a spherical homogeneous volume conductor.

- Preconditions
  - dipole in a fixed location
  - homogeneous spherical volume conductor



- Equilateral triangle

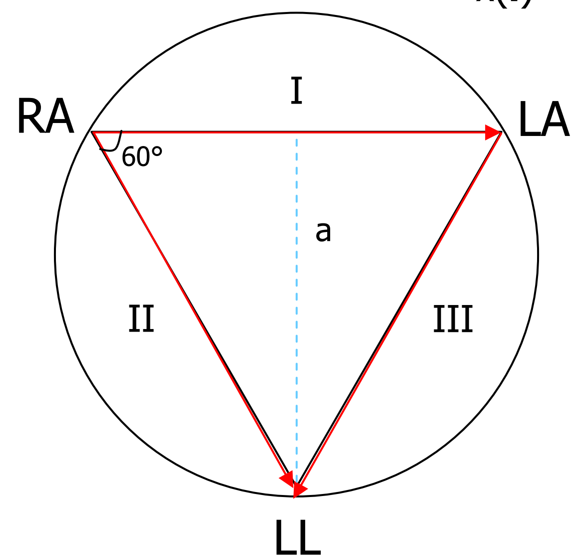
  $|C_I| = |C_{II}| = |C_{III}|$ (=1)

  $C_I = \mathbf{j}$

  $\sin 60° = |a| / |C_{II}| \implies |a| = \sin 60° = |C_{II}| = \sqrt{3}/2$

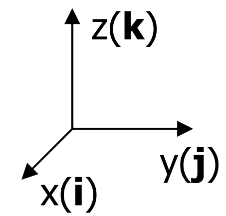
  $C_{II} = 1/2\,\mathbf{j} - \sqrt{3}/2\,\mathbf{k}$

  $C_{III} = -1/2\,\mathbf{j} - \sqrt{3}/2\,\mathbf{k}$



Q2: 12-lead ECG Lead Vectors

- Goldberger leads aVR, aVL ja aVF
- $|C_{aVR}| = |C_{aVF}| = |C_{aVL}| = |a| = \sqrt{3}/2$ (*|I|)



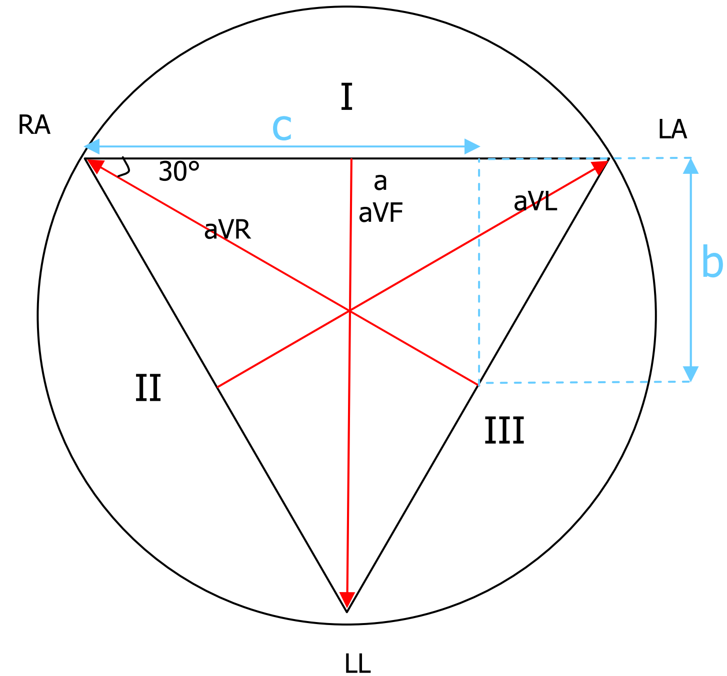
$C_{aVF} = -|a|\ \mathbf{k} = -\sqrt{3}/2\ \mathbf{k}$

$|c| = \cos 30° * |aVR| = \sqrt{3}/2 * \sqrt{3}/2 = 3/4$

$|b| = \sin 30° * |aVR| = 1/2 * \sqrt{3}/2 = \sqrt{3}/4$

$C_{aVL} = |c|\mathbf{j} + |b|\mathbf{k} = 3/4\ \mathbf{j} + \sqrt{3}/4\ \mathbf{k}$

$C_{aVR} = -|c|\mathbf{j} + |b|\mathbf{k} = -3/4 j + \sqrt{3}/4\ \mathbf{k}$



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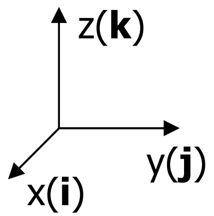
Q2: 12-lead ECG Lead Vectors



- Leads VR, VF and VL
- $|C_{VR}| = |C_{VF}| = |C_{VL}| = ?$

  $1/2*|C_I| = |C_{VR}|*\cos 30°$

  $|C_{VR}| = 1/2*|C_I| / \cos 30°$

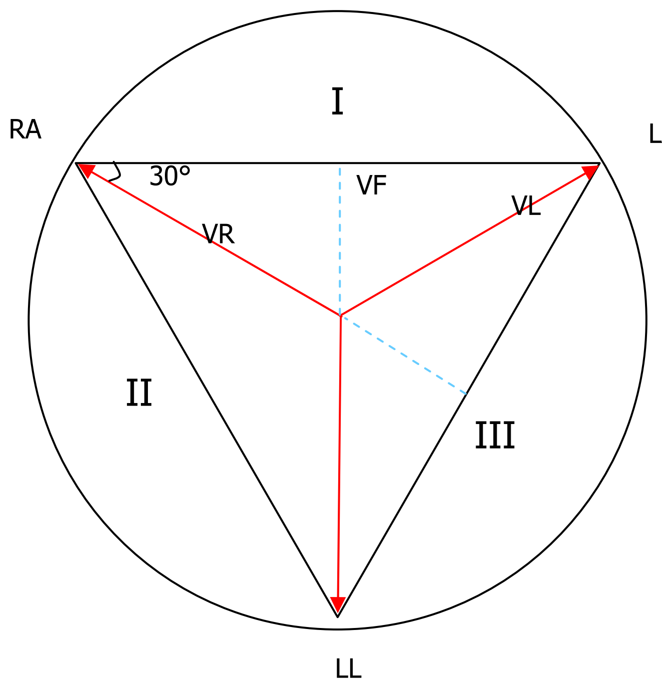
  $= 1/2 / \sqrt{3}/2 = 1/\sqrt{3}$

  $|C_{VF}|/|C_{aVF}| = 1/\sqrt{3} / \sqrt{3}/2 = 2/3$

  $C_{VF} = C_{aVF} * 2/3 = \mathbf{-}\sqrt{3}/2\ \mathbf{k} * 2/3 = -\sqrt{3}/3\ \mathbf{k}$

  $C_{VL} = C_{aVL} * 2/3 = (3/4\ \mathbf{j} \mathbf{+} \sqrt{3}/4\ \mathbf{k})* 2/3$
  $= 1/2\ \mathbf{j} + \sqrt{3}/6\ \mathbf{k}$

  $C_{VR} = C_{aVR} * 2/3 = (-3/4\ \mathbf{j} \mathbf{+} \sqrt{3}/4\ \mathbf{k})* 2/3$
  $= -1/2\ \mathbf{j} + \sqrt{3}/6\ \mathbf{k}$



Q3: 12-lead ECG Lead Vectors

- During a QRS-complex at some time instant t the following potentials were measured:

III-lead	+1,1 mV
aVR-lead	-3,4 mV
V1 -lead	-3,5 mV

  Approximate the potentials in the leads I, II, aVL, aVF, V6 and V4.
- We need **c** & **p** to solve the potentials (V=**c•p**)
- Frontal plane: III & aVR identify **p**

$|C_I| = 2/\sqrt{3}*|C_{aVR}| \Rightarrow V'_{aVR} = -3.93$ mV

$|\mathbf{p}| = \sqrt{(3.93^2 + 1.1^2)} = 4.1$ mV

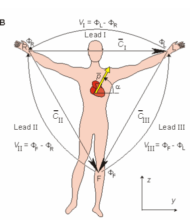
$\tan\alpha = 1.1/3.93 \Rightarrow \alpha = 15.6°$

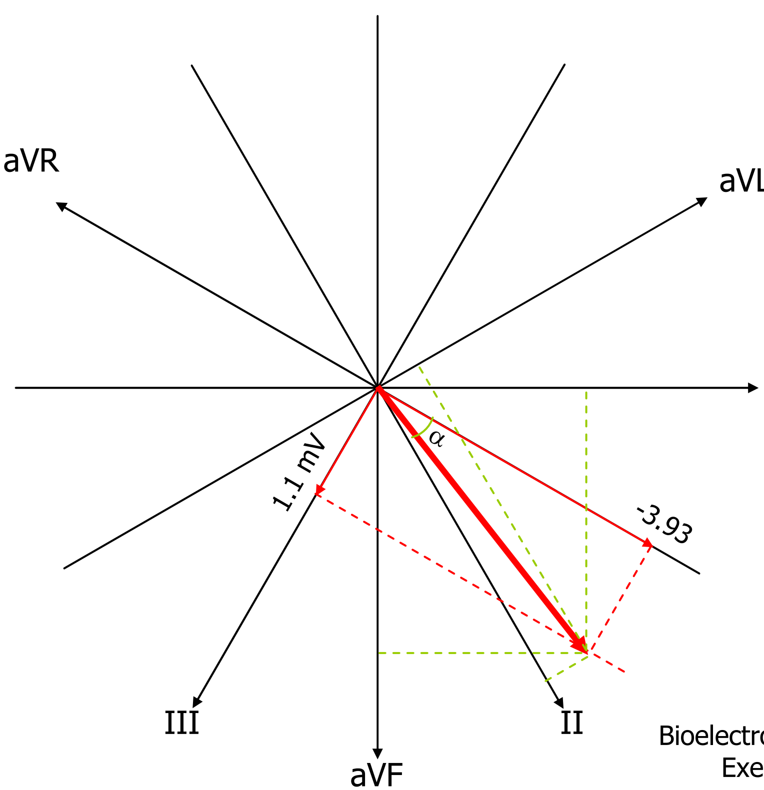
$V_I = 4.1 * \cos(30+\alpha) = 2.9$ mV

$V_{II} = 4.1 * \cos(30-\alpha) = 4.0$ mV

$V'_{aVL} = 4.1 * \cos(60+\alpha) = 1.0$ mV

$V'_{aVF} = 4.1 * \cos(60-\alpha) = 2.9$ mV





Augmented leads scaling:

$V_{aVL} = V'_{aVL} * \sqrt{3}/2 = 0.9$ mV

$V_{aVF} = V'_{aVF} * \sqrt{3}/2 = 2.5$ mV

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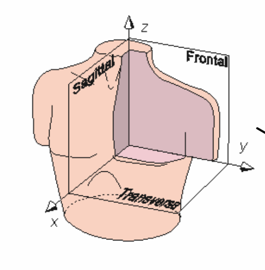
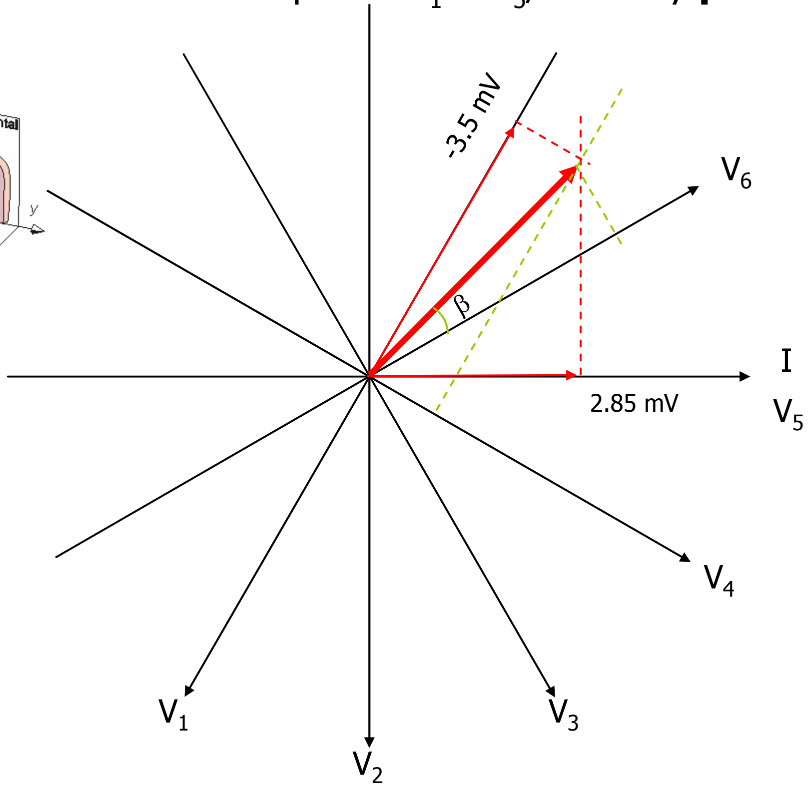
Q3: 12-lead ECG Lead Vectors

- Potentials V6 and V4?
- Transverval plane: V_1 & V_5/I identify **p**





V_{V1} = -3.5 mV
$V_I = V_{V5}$ = 2.85 mV

x*cos(30-β) = 3.5
x*cos(30+β) = 2.85
➔ β = 10.1° (HOW?, next slide)
➔ |**p**| = 2.8/cos(30+β) = 3.7 mV

V_{V6} = 3.6 * cos(β) = 3.7 mV
V_{V4} = 3.6 * cos(60+β) = 1.3 mV

assumption: $|c_I| = |c_{V1...6}|$

Q3: 12-lead ECG Lead Vectors

Summation equation: $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

x*cos(30-β) = 3.5 => cos30cosβ + sin30sinβ = 3.5/X
x*cos(30+β) = 2.85 => x = 2.85/(cos30cosβ-sin30sinβ)

cos30cosβ + sin30sinβ = 3.5/ 2.85 * (cos30cosβ-sin30sinβ)   (a=3.5/2.85)
cos30cosβ -1.25*cos30cosβ = -a*sin30sinβ - sin30sinβ

(cos30-a*cos30)*cosβ = -(a*sin30+sin30)*sinβ

sinβ/cosβ = -(cos30-a*cos30)/(a*sin30+sin30)

tan β = 0.1755   => β=10.1°

Q4: Lead vectors…

- On the outer rim of a two dimensional volume conductor at points P_i, i = 1,..., 6 the potentials generated by a unit dipole oriented parallel to X or Y axes at point P_o are as follows:

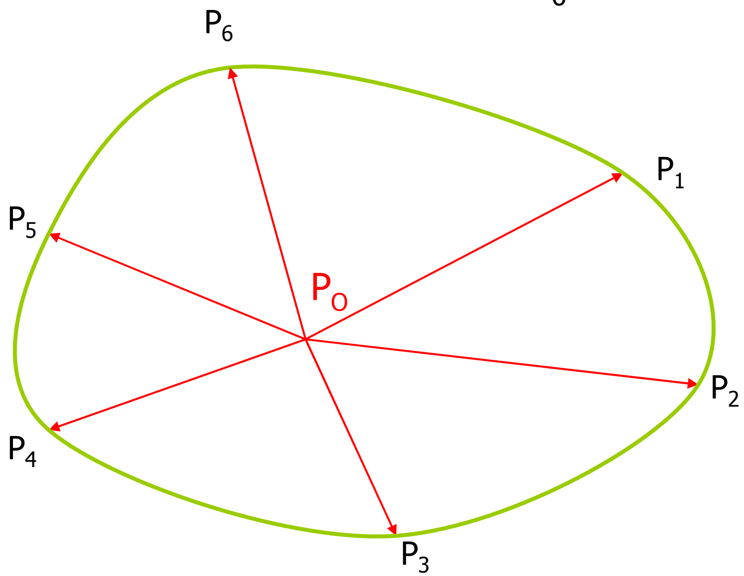
Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5

- Using these measurements is it possible to
  - a) Derive the image surface of this source dipole location?
  - b) Calculate the lead vector of a lead between the points 3 and 6?
  - c) Derive the potential at the point 4 generated by a unit dipole at a point Pz?
  - d) Derive the lead field of the volume conductor?
  - e) Construct a VECG lead system (X and Y-leads) that would measure the X and Y components of the dipole at the point Po with similar sensitivity?

Q4: Lead vectors…

- Is it possible to…
  - a) Derive the image surface of this source dipole location? yes

Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5



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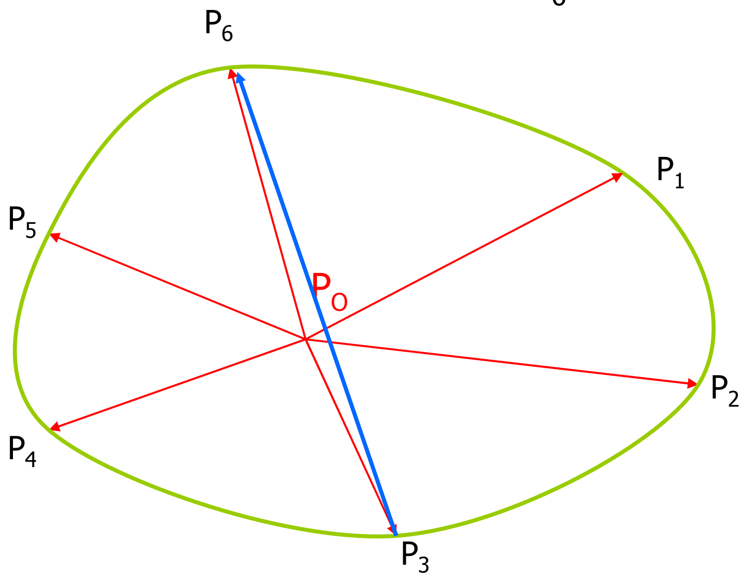
Q4: Lead vectors...

- Is it possible to...
  - b) Calculate the lead vector of a lead between the points 3 and 6?
    yes

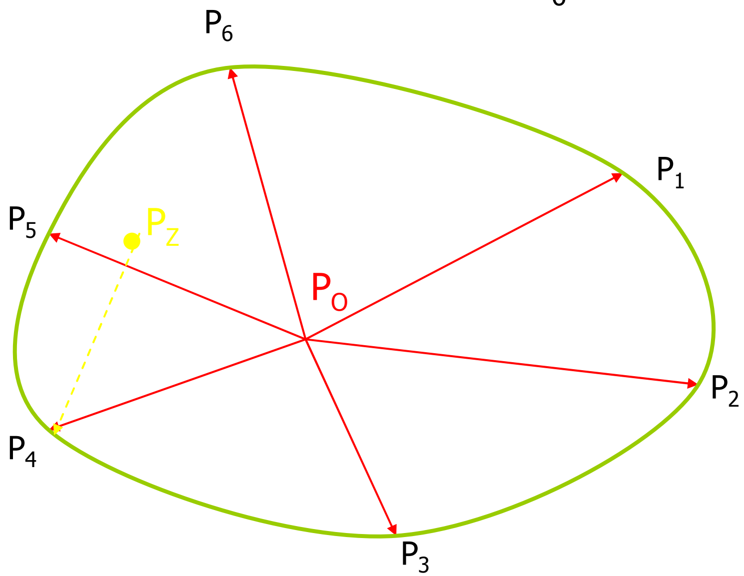
Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5



Q4: Lead vectors…

- Is it possible to…
  - c) Derive the potential at the point 4 generated by a unit dipole at a point Pz?
    no

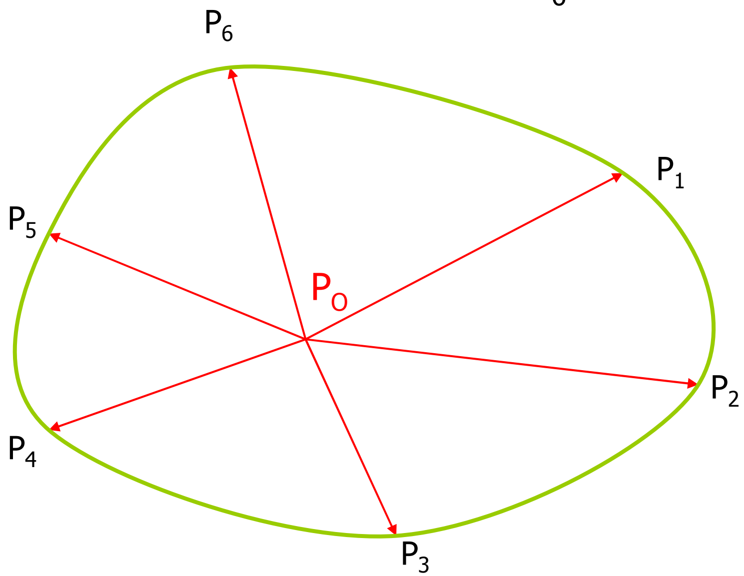
Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5



Q4: Lead vectors…

- Is it possible to…
  - d) Derive the lead field of the volume conductor?
    - no

Electrode P_i	Potentials generated by X and Y dipoles V_X	V_Y
1	4	5
2	6	-1
3	1	-4
4	-4	-2
5	-4	2
6	-1	5



Q4: Lead vectors…

- Is it possible to…
  - e) Construct a VECG lead system (X and Y-leads) that would measure the X and Y components of the dipole at the point Po with similar sensitivity? yes

Electrode	Potentials generated by X and Y dipoles	
P_i	V_X	V_Y
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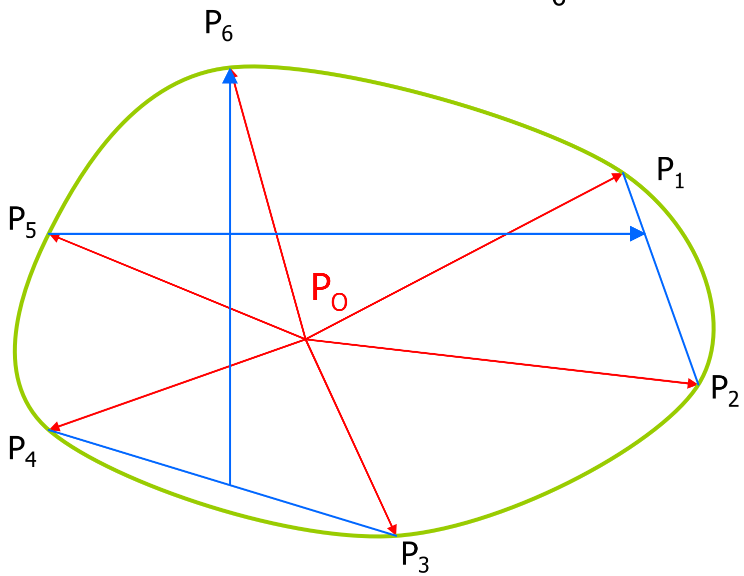


Q5: EEG Sensitivity

- Figure 1 represents a potential data distribution at a model of a (half) head generated by a reciprocal current of 1.0 μA applied to electrodes C and D. Calculate the potential between electrodes C and D generated by current dipoles A and B (|P| = 4 μAcm).

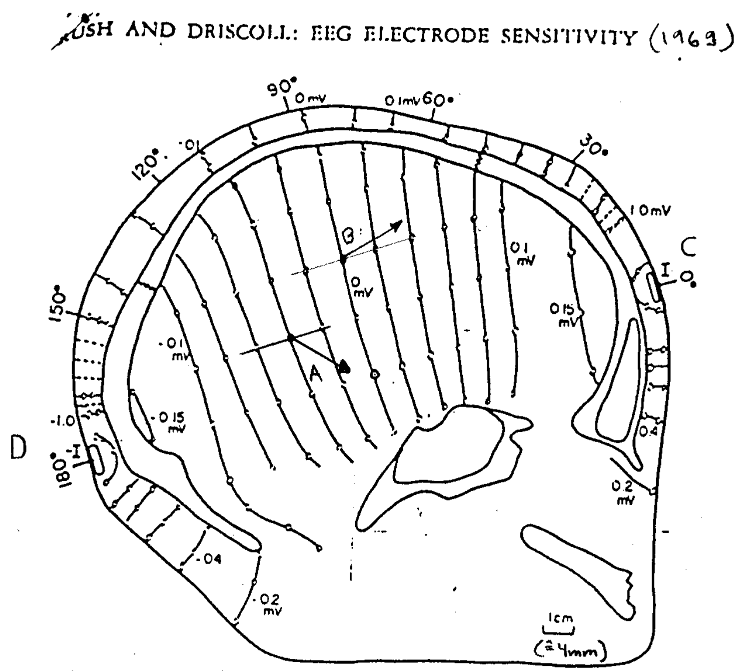


Fig 1. Potential data at surface of a electrolytic tank (midsagittal plane). Lines between 0.1 mV curves are spaced at equal potential differences (0.02 mV) apart. I=1.0 μA in the half-head model; fluid resistivity is 2220 Ωcm.

Q5: EEG Sensitivity

- Reciprocal current I_{CD} = 1.0 μA applied to electrodes C and D forms the lead field.
  - now shown as potential field
  - shows the sensitivity of lead CD to dipolar sources
  - distances of the lines = lead sensitivity
- "Golden equation" 11.29

source

$$V_{CD} = \frac{1}{I_{CD}} \int_v \frac{1}{\sigma} \bar{J}_{LE_{CD}} \bullet \bar{J}_i \, dv$$

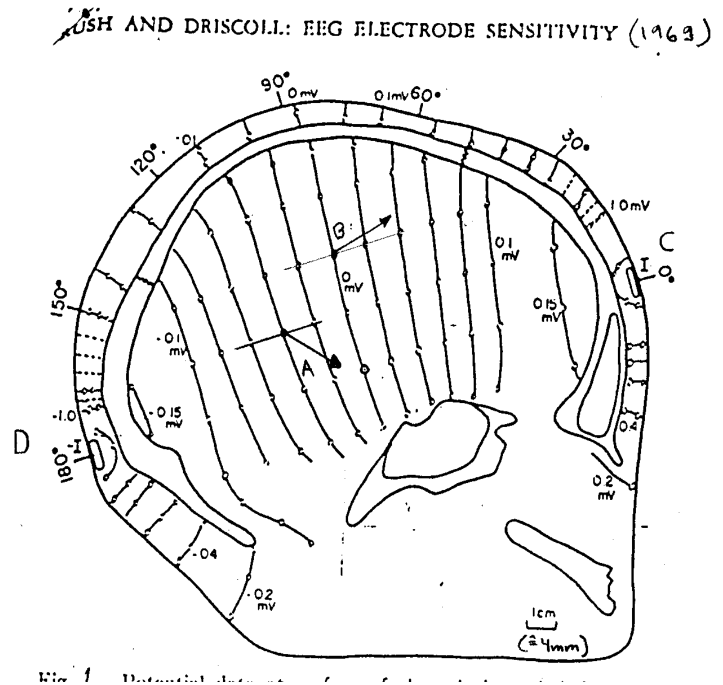
reciprocal lead field

  - convert to E-field form

$$\bar{J} = \sigma \bar{E}, \ \bar{E} = \frac{\Delta V}{\Delta x}$$

$$\Rightarrow \bar{J}_{LE_{CD}} = \sigma \frac{\Delta V}{\Delta x}$$

$$V_{CD} = \frac{1}{I_{CD}} \int_v \frac{\Delta V}{\Delta x} \bullet \bar{J}_i \, dv$$



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Q5: EEG Sensitivity

- One dipole source

$$V_{CD} = \bar{c} \bullet \bar{p} = |\bar{c}||\bar{p}| * \cos\alpha$$

$$\bar{c} = \frac{\bar{E}}{I_{CD}} = \frac{\Delta V}{I_{CD} \Delta x}$$

$$\bar{p} = 4\,\mu\text{Acm}$$

– $|\mathbf{c}_A| = \frac{0.04\,\text{mV}}{2.3\ \text{cm}} * \frac{1}{2\ \mu\text{A}} = 870\,\frac{\Omega}{m}$

$\alpha \approx 43°$ => $V_{CD} =$ 25 µV

– $|\mathbf{c}_B| = \frac{0.04\,\text{mV}}{2.45\ \text{cm}} * \frac{1}{2\ \mu\text{A}} = 820\,\frac{\Omega}{m}$

$\alpha \approx 17°$ => $V_{CD} =$ 31 µV

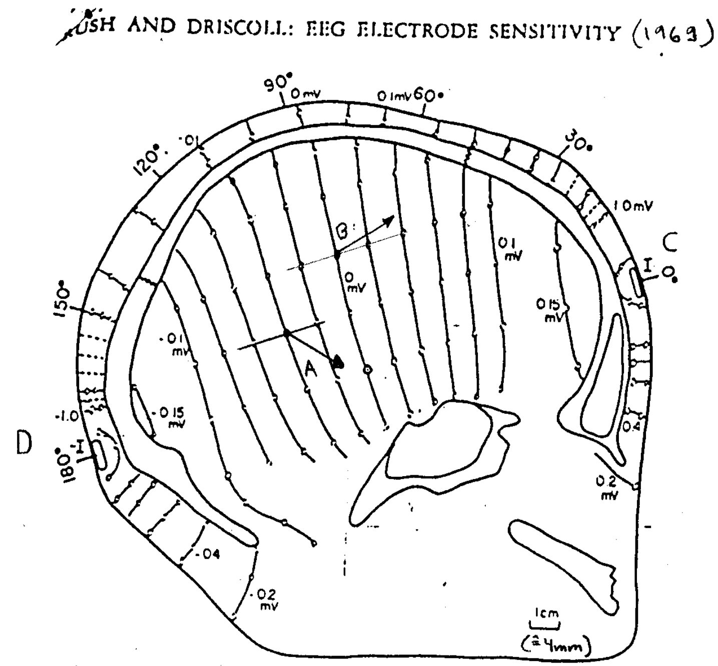

Fig 1. Potential data at surface of a electrolytic tank (midsagittal plane). Lines between 0.1 mV curves are spaced at equal potential differences (0.02 mV) apart. I=1.0 µA in the half-head model; fluid resistivity is 2220 Ωcm.